1992:

1) $f(x) = 3x^5 - 5x^3 + 2$

a) $f'(x) = 15x^4 - 15x^2$
$f'(x) = 0$
$15x^4 - 15x^2 = 0$
$15x^2(x^2 - 1) = 0$
$x = 0,\ x^2 = 1$
$x = \pm 1$

$f'(x)$: $+$ $-$ $-$ $+$ (at -1, 0, 1)

increasing on $(-\infty, -1)$ and $(1, \infty)$

b) $f''(x) = 60x^3 - 30x$
$f''(x) = 0$
$60x^3 - 30x = 0$
$30x(2x^2 - 1) = 0$
$x = 0,\ 2x^2 - 1 = 0$
$2x^2 = 1$
$x^2 = \frac{1}{2}$
$x = \pm \frac{1}{\sqrt{2}}$

$f''(x)$: $-$ $+$ $-$ $+$ (at $-1/\sqrt{2}$, 0, $1/\sqrt{2}$)

concave up on $\left(-\frac{1}{\sqrt{2}}, 0\right)$ and $\left(\frac{1}{\sqrt{2}}, \infty\right)$

c) horizontal tangent has slope 0
$f'(x) = 0$ when $x = 0, \pm 1$

when $x = 0$: $f(x) = 2$, $f'(x) = 0$
when $x = -1$: $f(x) = 4$, $f'(x) = 0$
when $x = 1$: $f(x) = 0$, $f'(x) = 0$

Equations of horizontal tangents:
$y - 2 = 0(x - 0)$ $\quad$ $y - 4 = 0(x + 1)$ $\quad$ $y - 0 = 0(x - 1)$
$y = 2$ $\quad$ $y = 4$ $\quad$ $y = 0$

2) $v(t) = 3(t-1)(t-3)$, $0 \le t \le 5$, $x(2) = 0$

a) $v(t) = (3t - 3)(t - 3) = 3t^2 - 9t - 3t + 9 = 3t^2 - 12t + 9$

$a(t) = 6t - 12$ $\quad$ a is increasing, so a is a minimum at $t = 0$

$a(0) = 0 - 12 = -12$ $\quad$ * or $a'(t) = 6 \neq 0 \therefore$ check endpoints *

b) Total distance $= \int_0^1 3(t-1)(t-3)\,dt - \int_1^3 3(t-1)(t-3)\,dt + \int_3^5 3(t-1)(t-3)\,dt$
$= \left[t^3 - 6t^2 + 9t\right]_0^1 - \left[t^3 - 6t^2 + 9t\right]_1^3 + \left[t^3 - 6t^2 + 9t\right]_3^5$
$= 4 - (-4) + 20$
$= 28$

c) Av. velocity $= \frac{1}{5-0}\int_0^5 (3t^2 - 12t + 9)\,dt = \frac{1}{5}\left[t^3 - 6t^2 + 9t\right]_0^5 = \frac{1}{5}(20) = 4$

3) $f(x) = \ln\left|\dfrac{x}{1+x^2}\right|$

a) Domain: $\dfrac{x}{1+x^2} \neq 0$

$x \neq 0 \qquad 1+x^2 \neq 0$

$\qquad\qquad x^2 \neq -1$

$x \neq 0$

b) $f(-x) = \ln\left|\dfrac{-x}{1+(-x)^2}\right| = \ln\left|\dfrac{-x}{1+x^2}\right| = f(x)$

since $f(-x) = f(x)$ function is even

c) $f'(x) = \dfrac{(1+x^2)(1) - x(2x)}{(1+x^2)^2} \div \dfrac{x}{1+x^2}$

$= \dfrac{-x^2+1}{(1+x^2)^2} \cdot \dfrac{1+x^2}{x}$

$= \dfrac{1-x^2}{x(1+x^2)}$

$f'(x) = 0$

$\dfrac{1-x^2}{x(1+x^2)} = 0$

$1-x^2 = 0, \quad x(1+x^2) = 0$

$x^2 = 1 \qquad x = 0,\ 1+x^2 = 0$

$x = \pm 1 \qquad\quad x^2 \neq -1$

$f'(x)$: $+$ -1 $-$ 0 $+$ 1 $-$

Max at $x=-1$ since $f'(x)$ changes sign from +ve to -ve and Max at $x=1$ since $f'(x)$ changes sign from +ve to -ve

d) $\lim\limits_{x\to 0^+} f(x) = \ln 0 = -\infty$ $\qquad \lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \ln\left|\dfrac{1/x}{1/x^2+1}\right| = \ln 0 = -\infty$

$\lim\limits_{x\to 0^-} f(x) = -\infty$ $\qquad \lim\limits_{x\to -\infty} f(x) = -\infty$

Max: $f(1) = \ln\dfrac{1}{2}$

$\therefore$ range $= \left(-\infty, \ln\dfrac{1}{2}\right]$

4) $y + \cos y = x + 1, \quad 0 \leq y \leq 2\pi$

a) $\dfrac{dy}{dx} - \sin y\dfrac{dy}{dx} = 1$

$\dfrac{dy}{dx}(1-\sin y) = 1$

$\dfrac{dy}{dx} = \dfrac{1}{1-\sin y}$

b) vertical tangent has no slope

$1 - \sin y = 0$

$\sin y = 1$

$y = \dfrac{\pi}{2}$

$\dfrac{\pi}{2} + \cos\dfrac{\pi}{2} = x + 1 \Rightarrow x = \dfrac{\pi}{2} - 1$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{1-\sin y}\right)$$

$$= \frac{(1-\sin y)(0) - (1)(-\cos y)\,dy/dx}{(1-\sin y)^2}$$

$$= \frac{\cos y \frac{dy}{dx}}{(1-\sin y)^2}$$

$$= \frac{\cos y\left(\frac{1}{1-\sin y}\right)}{(1-\sin y)^2}$$

$$= \frac{\cos y}{(1-\sin y)^3}$$

5)

a) $f(x) = e^{-x}$, $g(x) = kx$

$f'(x) = -e^{-x}$, $g'(x) = k$

$-e^{-x} = k$

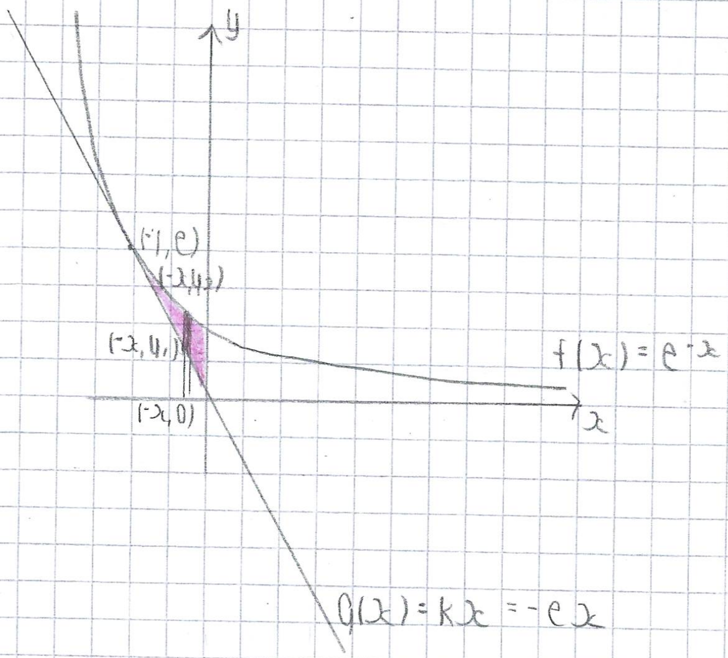
Also $e^{-x} = kx$

$e^{-x} = -e^{-x}x$

$x = -1$

$k = -e^{-(-1)} = -e^{1} = -e$

$x = -1, k = -e$



b) $\int_{-1}^{0} (e^{-x} - (-ex))\,dx$

$= \int_{-1}^{0} (e^{-x} + ex)\,dx$

$= \frac{e}{2} - 1$

c) $R(x) = y_2 - 0 = e^{-x}$

$r(x) = y_1 - 0 = -ex$

$$\text{Volume} = \pi\int_{-1}^{0}\left((e^{-x})^2 - (-ex)^2\right)dx$$

6)

a) $\frac{dV}{dt} \propto \frac{1}{r}$ $\qquad V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{k}{r}$ $\qquad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\therefore \frac{k}{r} = 4\pi r^2 \frac{dr}{dt}$

$k = 4\pi r^3 \frac{dr}{dt}$

$\int k\,dt = \int 4\pi r^3\,dr$

$kt + c = \pi r^4$ $\qquad$ When $t = 0, r = 1$

$0 + c = \pi$

$c = \pi$

$\therefore kt + \pi = \pi r^4$

when $t = 15$, $r = 2$

$15k + \pi = 16\pi$

$15k = 15\pi$

$k = \pi$

$\therefore \pi t + \pi = \pi r^4$

$\pi r^4 = \pi(t+1)$

$r^4 = t+1$

$r = \sqrt[4]{t+1}$

b) When $t = 0$: $V = \frac{4}{3}\pi(1) = \frac{4}{3}\pi$

$\frac{4}{3}\pi r^3 = 27\left(\frac{4}{3}\pi\right)$

$\frac{4}{3}\pi r^3 = 36\pi$

$r^3 = 27$

$r = 3$

$3 = \sqrt[4]{t+1}$

$t+1 = 3^4$

$t = 3^4 - 1$

$t = 81 - 1 = 80$